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DISCUSSION OF PROCEEDINGS - SEPARATES

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DISCUSSION OF RELIEF WELL SYSTEMS FOR DAMS AND LEVEES PROCEEDINGS-SEPARATE NO. 192

P. T. BENNETT,[9] M. ASCE.—The four sand models described by the authors

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corroborate their statement that many combinations of conditions exist which cannot be covered completely by any single theory for the design of relief wells.

In models representing specific local conditions, such as model C and model D, an attempt to draw generalized conclusions from the results is unnecessary because the conclusions can be applied only to the prototype. It is highly desirable whenever possible to obtain quantitative data from generalized models such as model A and model B.

Interpretation of model-study results and their application is vastly simplified by separating the effects of well spacing, well diameter, and depth of penetration from the effects of relatively remote boundary conditions (such as the distance to the effective source of seepage, and the conditions of flow landward of the line of wells) of the seepage system. The possibilities of such a division of the whole system into component parts are illustrated in Figs. 13 to 17.

In model B no seepage was permitted to escape on the landside of the wells. The pressure gradient consequently approaches zero, and the "effective landside exit distance" is infinite for all three cases shown in Fig. 13. The "effective riverside distance" can be obtained graphically from Fig. 13 by drawing a tangent to the pressure-gradient curves at the points of inflection between the line of wells and the open borrow pit. This riverside tangent and the corresponding landside tangent intersect at the line of wells at a pressure defined as the average head in the line of wells. Extension of the riverside tangent gradient riverward to intersect the line representing the 100% head establishes the effective riverside distance. These tangents result in effective riverside distances of approximately 1,000 ft, 700 ft, and 560 ft for models B-a, B-b, and B-c, respectively.

These effective distances are defined by the pressure gradients at the top of the sand stratum. From Fig. 17 it can be seen that an entirely different set of gradients and effective distances would be obtained from piezometers placed at the bottom or midpoint depth in the sand. However, if it is assumed that the flow through the entire depth of the foundation is proportional to the gradient at the elevation indicated by Fig. 13, then

$$Q_w = C\,A\,(S_r - S_l) \qquad (3)$$

in which Q_w denotes the flow from the well, C is an unknown constant, A denotes the well spacing, S_r is the riverside gradient, and S_l represents the landside gradient. For model B, S_l is zero in all three cases. If the mean head in the line of wells divided by the entire depth of the formation is designated as P_a, the total head by P_t, and the effective riverside distance by L_r—

$$S_r = \frac{P_t - P_a}{L_r} \qquad (4)$$

Combining Eqs. 3 and 4 results in

$$\frac{Q_w L_r}{A (P_t - P_a)} = C \quad\text{.......................}\quad (5)$$

Eq. 5 is completely independent of any dimensions of the well system since Q_w/A is the seepage per linear foot and the well spacing A is introduced because the flow in Figs. 14 to 16 is in terms of Q_w. Consequently, if the experimental data fit Eq. 5 with reasonable accuracy, the flow per unit length from the line of wells is directly related to the exterior boundary conditions, and the relation is reasonably independent of the size, spacing, and penetration of the wells.

Table 2 seems to substantiate the previous assumptions. In Table 2, the designation "a-58-25" indicates that test data from model B-a with a well spacing of 58 ft and a penetration of 25%, has been taken from Figs. 14 to 16 to compute the assumed constant C from Eq. 5.

TABLE 2.—EVALUATIONS OF CONSTANTS C AND C_n FROM MODEL DATA

Test	C, in gal per min per lin ft	C_n, in gal per min per sq ft	Test	C, in gal per min per lin ft	C_n, in gal per min per sq ft
a-58-10	72	0.068	a-58-50	70	0.39
b-58-10	67	0.058	b-58-50	70	0.34
c-58-10	70	0.068	c-58-50	67	0.40
Mean		0.065	Mean		0.38
a-87-10	70	0.048	a-87-50	73	0.24
b-87-10	64	0.042	b-87-50	69	0.23
c-87-10	66	0.049	c-87-50	69	0.23
Mean		0.046	Mean		0.23
a-174-10	69	0.018	a-174-50	80	0.15
b-174-10	52	0.020	b-174-50	68	0.15
c-174-10	74	0.020	c-175-50	74	0.16
Mean		0.019	Mean		0.15
a-58-25	68	0.16	a-58-100	74	1.7
b-58-25	75	0.14	b-58-100	70	1.6
c-58-25	64	0.094	c-58-100	68	1.3
Mean		0.13	Mean		1.5
a-87-25	73	0.11	a-87-100	76	1.4
b-87-25	70	0.086	b-87-100	60	0.87
c-87-25	67	0.092	c-87-100	64	1.0
Mean		0.096	Mean		1.1
a-174-25	76	0.064	a-174-100	75	0.75
b-174-25	69	0.053	b-174-100	66	0.72
c-174-25	65	0.049	c-174-100	66	0.72
Mean		0.055	Mean		0.73

Considering the dimensions of the well system as opposed to the general relations, between the system and the remote boundary conditions, it can be assumed that

$$\frac{Q_w}{A} = C_n P_a \quad\text{.........................}\quad (6)$$

in which C_n is a coefficient presumed to be constant for a given well installation and is independent of the precise nature of the exterior or remote boundary conditions. The tabular values (Table 2) compiled from the data in Figs. 14 to 16 appear to substantiate Eq. 6.

The development of well-system formulas need not be complicated by introducing the possible exterior boundary conditions in the field. Also, the effect of any drainage system on a given set of field conditions can be studied without going into the details of the particular set of wells to be provided. (This statement must be qualified if the length of the drainage system is short, as in the case of a few wells.)

The electric analogy model studies described by Messers. Middlebrooks and Jervis[5] also can be taken as an illustration of this principle. These studies were made with exterior boundary conditions similar to those in model A, but the results were reported in a form facilitating the entire elimination of exterior boundary conditions. In the writer's discussion of the paper by Messrs. Middlebrooks and Jervis, it was shown that the effective distances to the source and landside exit, and the heads at source and exit, can be eliminated or generalized by substitution of the riverside and landside seepage gradients which determine the flow of the wells.[10] This elimination of specific exterior boundary conditions involved the substitution of the concept of a mean head in the line of wells for the "extra length" concept of Messrs. Middlebrooks and Jervis. There is no essential difference between these two concepts since the mean head P_a (referred to head at well) and the extra length L_x have the simple relation:

$$L_x\ (S_r - S_l) = P_a \qquad (7)$$

The formula developed by Mr. Barron[6] for finite effective landside and riverside distances can also be transformed into an identity with the extra length or mean pressure formulas for fully penetrating wells. For partial penetration wells, no purely analytic formula is available, and the charts of Messrs. Middlebrooks and Jervis have been the only available guide. The use of these charts involves double interpolation (with respect to D/A and penetration), and for that reason they have not been used to full advantage. The term D is used to denote the depth of the pervious stratum.

The data of Messrs. Middlebrooks and Jervis were re-evaluated in the hope that some method could be developed to facilitate interpolation for any value of D/A and percentage of penetration. As the first step in the re-evaluation, the extra-length concept was replaced by the mean-head approach, so that the two sets of original curves[5] showing the "extra length" and the midpoint head might be replaced by a single set of curves involving midpoint head and mean heads only. Model data for these heads were replotted against the logarithm of A/R_w, as shown in Fig. 20. The term R_w denotes the radius of the well. For the penetrations shown in Fig. 20 the ratio D/A is equal to 3.

[5] "Relief Wells for Dams and Levees," by T. A. Middlebrooks and W. H. Jervis, *Transactions*, ASCE, Vol. 112, 1947, p. 1321.

[10] Discussion by P. T. Bennett of "Relief Wells for Dams and Levees," by T. A. Middlebrooks and W. H. Jervis, *Transactions*, ASCE, Vol. 112, 1947, p. 1376.

[6] "The Effect of a Slightly Pervious Top Blanket on the Performance of Relief Wells," by R. A. Barron, *Proceedings*, Second International Conference on Soil Mechanics, Vol. IV, 1948.

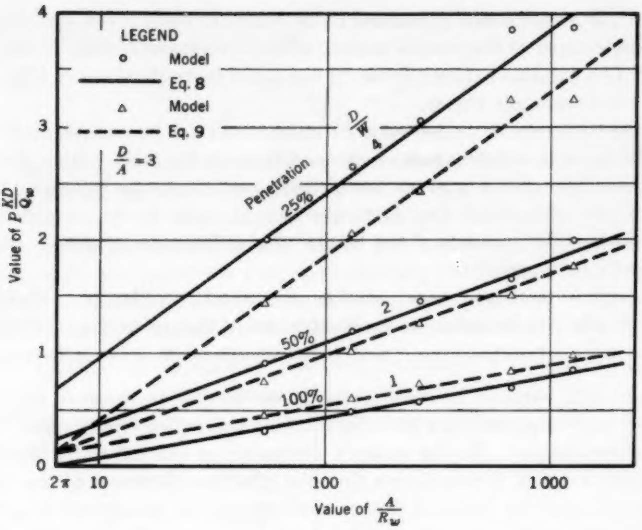

FIG. 20.—COMPARISON OF EMPIRICAL EQUATIONS WITH MODEL TEST DATA

In drawing lines through the experimentally determined data, a considerable degree of individual judgment is involved. It appeared that the semi-log plots defined nearly straight lines, and that the lines for midpoint and mean heads were nearly parallel for a given well installation. Analytic support for the interpretations of these lines as straight and parallel can be stated as follows:

1. In the region of the wells, the flow is composed of radial components toward the wells and a linear component normal to the line of wells. Only the radial components contribute to the flow and drawdown of the wells, and the other components can be neglected.

2. In the immediate vicinity of any one well, the radial components caused by other wells are quite small, and the flow to the well in question can be considered to be purely radial to it. For radial flow, the pressure is proportional to the logarithm of the radial distance.

3. For two or more well systems having all dimensions identical (except for the radius of the well) and having identical well flow, the pressure distribution will be identical except that, as the well radius becomes smaller, an increment of head loss proportional to an increment of $\log_e R_w$ will be added. This condition is illustrated in Figs. 21 and 22.

4. For successive systems which are identical or geometrically similar except for well radius, the difference between the mean and midpoint heads is constant, and the difference between the midpoint—or mean—head and the head at the well varies as $\log_e R_w$. The assumption of straight and parallel plots of pressure versus $\log_e (A/R_w)$ is therefore justified.

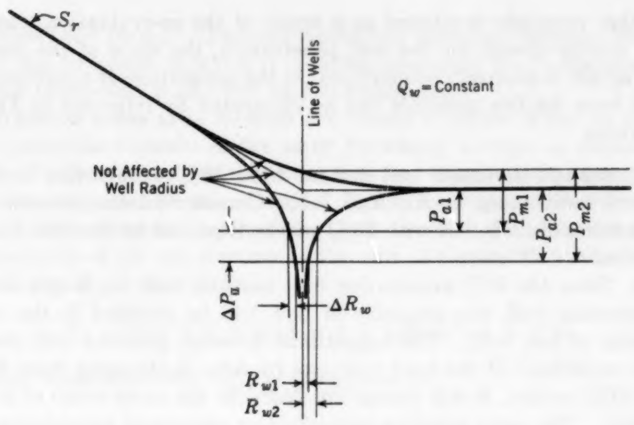

FIG. 21.—EFFECT OF WELL RADIUS ON PRESSURE DISTRIBUTION

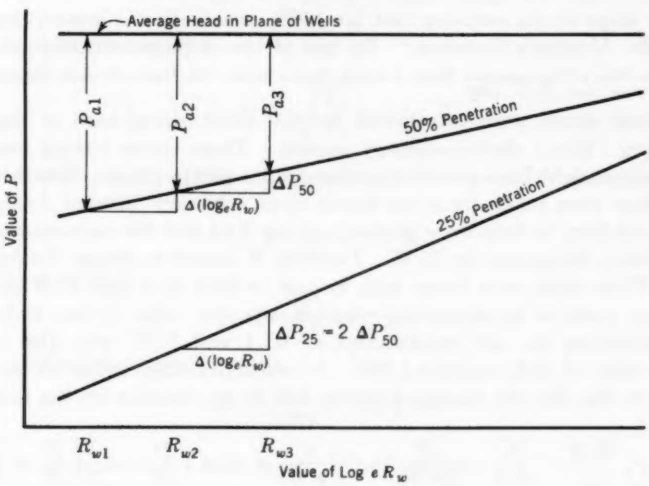

FIG. 22.—EFFECT OF WELL PENETRATION AND RADIUS ON PRESSURE DISTRIBUTION

Another principle developed as a result of the re-evaluation was that, for systems similar except for the well penetration, the slope of the straight-line plots (Fig. 22) is inversely proportional to the percentage of penetration. The physical basis for this principle can be illustrated by reference to Fig. 22 and the following:

a. Assume a certain well system with 50% penetration and a series of well radii equal to R_1, R_2, ... Consider another system with the same series of radii, but with 25% penetration, and let the well discharge be identical for all cases.

b. Since the 25% penetration well has just half the length of the 50% penetration well, the intensity of flow will be doubled in the immediate vicinity of the well. The logarithmic pressure gradient will therefore be twice as steep. If the head increases by ΔP_{50} in changing from R_2 to R_3 in the 50% system, it will change by $2\Delta P_{50}$ in the same interval in the 25% system. The same principle holds for any amount of penetration provided the permeability does not change with depth.

It should be noted that these analytic principles do not define the absolute values of the midpoint and mean heads. However, with absolute values experimentally available, these principles assist materially in the process of smoothing out observational irregularities.

The slope of the semi-log plot for 100% penetration is known analytically from Mr. Muskat's formulas.[4] By use of the slope-penetration relationship theoretical slopes were determined for the penetrations used in the original Vicksburg (Miss.) electric-analogy models. These slopes having been predetermined, straight lines were drawn through the plotted data. The intercepts of these lines were taken from the charts at an arbitrary value of A/R_w equal to 1,000, and they in turn were plotted against D/A and the reciprocal of percent penetration, designated by D/W. The term W is used to denote the depth of the well. These plots were linear with respect to both D/A and D/W, and it was therefore possible to interpolate algebraically the value of the midpoint and mean pressures for any combination of D/A and D/W with the arbitrarily chosen value of A/R_w equal to 1,000. In lieu of drawing a set of curves on charts similar to Fig. 20, the family of curves can be represented by the following:

$$P_a \frac{K D}{Q_w} = \frac{P_a}{A S} = \frac{D}{2 \pi W} \log_e \frac{A}{2 \pi R_w} + 0.11 \left(\frac{D}{A} - 1\right)\left(\frac{D}{W} - 1\right) \ldots (8)$$

$$P_m \frac{K D}{Q_w} = \frac{P_m}{A S} = \frac{D}{2 \pi W} \log_e \frac{A}{2 \pi R_w} + 0.11 \ldots\ldots\ldots\ldots (9)$$

in which S equals the algebraic sum of the mean gradients normal to the line of wells, P_m is the head midway between wells at the top of the sand stratum, and $0.11 = \log_e 2/(2\pi)$. Eqs. 8 and 9 are empirical, even though certain analytic principles were used in their formulation. It should be noted that the equations are derived from the early basic work reported by Messrs.

[4] "The Flow of Homogeneous Fluids Through Porous Media," by Morris Muskat, McGraw-Hill Book Co., Inc., New York, N. Y., 1937.

Middlebrooks and Jervis.[5] The only significant difference between the original work and the new form in which it is herein presented lies in the greater ease of accurate interpolation afforded by the equations.

The question arises as to whether the results of model B can be correlated with the generalized results of the early Vicksburg models, as stated in Eqs. 8 and 9. To investigate this question, it is first necessary to transform the nonhomogeneous sand strata of model B into an equivalent homogeneous system. Taking the total depth of the three strata as the unit of length, and the permeability of the top stratum as the unit of permeability, the horizontal transmissibility T_h of the system is

$$T_h = 0.25 \times 1 + 0.25 \times 2 + 0.50 \times 6.4 = 3.95 \qquad (10a)$$

The vertical transmissibility (T_v) of the system is

$$T_v = \frac{1}{\frac{0.25}{1} + \frac{0.25}{2} + \frac{0.50}{6.4}} = 2.2 \qquad (10b)$$

The transformed homogeneous permeability both vertically and horizontally is

$$K_t = \sqrt{T_h T_v} = 2.96 \qquad (10c)$$

and the transformed depth is

$$D_t = D \sqrt{\frac{T_h}{T_v}} = 1.34\, D \qquad (10d)$$

The transformed depths of the individual strata become (1) for the top—

$$25 \times \frac{1}{2.96} = 8.5 \text{ ft} \qquad (11a)$$

(2) for the middle—

$$25 \times \frac{2}{2.96} = 17.0 \text{ ft} \qquad (11b)$$

and (3) for the bottom—

$$50 \times \frac{6.4}{2.96} = 108.0 \text{ ft} \qquad (11c)$$

resulting in a total transformed depth (D') of 133.5 ft.

Corresponding to the authors' percent penetration expressed in depth, the effective penetrations based on transmissibility are as follows:

Depth, in percent	Transmissibility, in percent
10	2.54
25	6.35
50	19.0
100	100

Transformed ratios of depth to well spacing were obtained by using a transformed total depth of 134 ft.

By using the transformed dimensions of model B, it is possible to compute theoretical values for $\frac{P_a}{A\,S}$ from Eq. 7, and also the observed values obtained from the model. A comparison of the observed and computed values is shown in Table 3. The underlined value is computed from the empirical formula.

TABLE 3.—THEORETICAL AND OBSERVED VALUES OF $\frac{P_a}{A\,S}$

Well spacing A, in feet	Value of $\frac{D'}{A}$	Model	Percent Penetration (Depth)			
			10	25	50	100
			Percent Penetration (Transmissibility)			
			2.54	6.35	19.0	100
29	4.6		25.4	9.8	3.0	0.24
		B-a	21.1	9.7	4.0	0.7
		B-b	24.7	13.3	3.9	1.0
		B-c	19.7	11.8	3.7	0.9
58	2.3		19.8	7.8	2.5	0.35
		B-a	17.9	7.4	3.0	0.5
		B-b	19.7	9.5	3.0	0.7
		B-c	16.3	7.5	2.7	0.7
87	1.54		19.1	7.6	2.5	0.42
		B-a	17.3	8.0	3.3	0.6
		B-b	17.3	9.1	3.4	0.8
		B-c	16.5	8.5	3.4	0.7
174	0.77		20.2	8.1	2.7	0.53
		B-a	21.6	6.9	3.1	0.5
		B-b	14.5	7.8	2.7	0.5
		B-c	22.5	7.5	2.6	0.5

The correlation between computed and observed data is quite satisfactory, considering that (1) basic data were taken visually from charts, (2) the foundation gradients are not uniform with depth, and (3) the transformation methods for obtaining effective penetrations and (D/A)-ratios are approximations. It would be interesting to compare computed midpoint heads with model results.

It is felt that the results of the authors' model tests strongly support the writer's contention that the characteristics of a well system can be determined independently of the over-all seepage system, and vice-versa. The results of the model studies performed by Messrs. Middlebrooks and Jervis, when expressed as Eqs. 7 and 8, appear to be applicable to nonuniform foundation conditions after transformation of the latter to an equivalent uniform condition. For that reason, in future model and analytic studies, attention might well be centered on those exterior features of the system which determine the effective distance to the "source" or "landside exit."

Model B illustrates the possibilities in the field. Fig. 13 shows the effective upstream resistance for model B-b to be 700 ft which can be considered as composed of the 350 ft from the line of wells to the center of the borrow pit, plus the equivalent of 350 ft involved in the vertical entrance through the bor-

row pit. In model B-c, this equivalent resistance through the borrow pit floor is in "parallel" with the 650 ft of resistance from the borrow pit to the open face at the river, and these two are in "series" with the 350 ft between the borrow pit and the wells. The total "resistance" of model B-c, computed from these partial resistances, is $\frac{1}{1/350 + 1/650} + 350 = 230 + 350 = 580$ ft, which closely approximates the effective riverside distance obtained experimentally. Following the experimental determination of the effective resistance of the borrow pit entrance, the total resistance of a system with such a pit at any location could be estimated by these methods.

Thus, it appears that generalized conclusions can be drawn from model studies of typical exterior systems—entirely independent of the particular drainage system used. Such general results, combined with those available for relief-well systems, should provide the method of attacking a variety of problems with a minimum of model studies.

Real progress in the field of relief-well design is also dependent on obtaining reasonably accurate basic information on the stratification of the foundation and top strata, with respect to permeability. Field permeability tests and observation of existing well systems are the best approach to this basic problem.

JOHN A. FOCHT, JR.,[11] J.M. ASCE.—A definite need has always existed for verification of theoretical methods of analysis of relief-well systems at locations where the landside top stratum is semipervious. Although reference was made by the authors to the approximate solutions which have been developed, no comparison was made between the model results and the theoretical analyses.

[11] Associate Engr., Greer and McClelland, Cons. Foundation Engrs., Houston, Tex.

Mr. Barron developed approximate formulas which can be used to compute the pressure midway between fully penetrating wells and the well flows where there is a semipervious, landside top stratum.[6] The pressure beneath the top stratum midway between partially penetrating wells can be computed approximately by applying a correction factor to the pressure computed for fully penetrating wells. This correction factor is the ratio of the head midway between partially penetrating wells (computed from the curves developed by Messrs. Middlebrooks and Jervis[12]) to the head midway between fully penetrating wells for an impervious, landside top stratum. This correction factor is approximate but, as will be shown, yields results comparable to those indicated by the models.

[6] "The Effect of a Slightly Pervious Top Blanket on the Performance of Relief Wells," by R. A. Barron, *Proceedings*, Second International Conference on Soil Mechanics, Vol. IV, 1948.

[12] "Relief Wells for Dams and Levees," by T. A. Middlebrooks and W. H. Jervis, *Transactions*, ASCE, Vol. 112, 1947, p. 1333, Figs. 8(*a*) and 8(*b*).

Mr. Bennett also has developed an approximate solution for fully or partially penetrating wells in which the landside top stratum is semipervious. Mr. Bennett's method of analysis[13] will yield the average pressure in the plane

[13] Discussion by P. T. Bennett of "Relief Wells for Dams and Levees," by T. A. Middlebrooks and W. H. Jervis, *ibid.*, pp. 1378–1382.

of the wells and the well flows. The ratio of the maximum pressure midway between the wells to the average pressure in the plane of the wells for an impervious top stratum can be computed as a ratio between the curves determined by Messrs. Middlebrooks and Jervis.[12] Application of this ratio to the computed average pressure will indicate the maximum pressure midway between wells. This ratio can be computed and applied as a correction for either partially or fully penetrating wells.

It is of interest to compare the data obtained from Model A-a-2 (Fig. 10) with results obtained by theoretical analyses to prove or disprove the validity of the theoretical analyses. Therefore, the permeability and thickness of the prototype top stratum (Fig. 3) and the prototype well radius were used in the procedures outlined by Messrs. Barron and Bennett to obtain the maximum head midway between wells for various well spacings and penetrations. The computed maximum head midway between wells for 50% and 100% penetration wells is shown in Fig. 23 for different well spacings. Also shown are the observed model data taken from Fig. 10. In addition, a curve indicating the maximum pressure between wells, which would exist if the landside top stratum were impervious, has been plotted in Fig. 23. A similar group of curves for 25% penetration wells is given in Fig. 24.

Good agreement between both theoretical curves and the model data is indicated for fully penetrating wells. It should be noted that in Figs. 23 and 11, for small well spacings, both the model data and Mr. Barron's solution show a head between wells greater than that indicated for an impervious top stratum. This condition is one which cannot exist because the pressure at any location where the landside top stratum is semipervious must be less than that for an impervious top stratum.

For 50% penetration wells, there is more divergence between the model data and the theoretical results. A considerable difference is indicated for the wells penetrating 25% into the pervious stratum. An inconsistency is evident in Fig. 24 as the pressures computed from Mr. Barron's solution for large well spacing are indicated to be greater than the pressure without any wells. The inherent error introduced in the correction factor for the small partial penetration results in a computed pressure greater than that which can actually occur. The average pressure in the line of wells for 25% penetration, as computed by Mr. Bennett's procedure (which is not shown in Fig. 24), is comparable to the model data for pressure midway between wells.

From a study of Figs. 23 and 24, it can be concluded that either Mr. Barron's procedure or Mr. Bennett's procedure will indicate satisfactory pressures midway between wells as compared to the model data for well penetrations between 50% and 100%. However, inasmuch as both the model data and results from Mr. Barron's analysis show (for small well spacings) pressures greater than those indicated for an impervious top stratum and, inasmuch as the pressures computed by Mr. Barron's analysis for large well spacings and small penetration may be greater than the pressure without any wells, the solution as outlined by Mr. Bennett—extended to obtain the maximum pressure between wells—appears preferable for numerical solutions. If a well system is to be designed on the basis of the average pressure in the line of wells, the method originally suggested by Mr. Bennett will be satisfactory. A comparison which

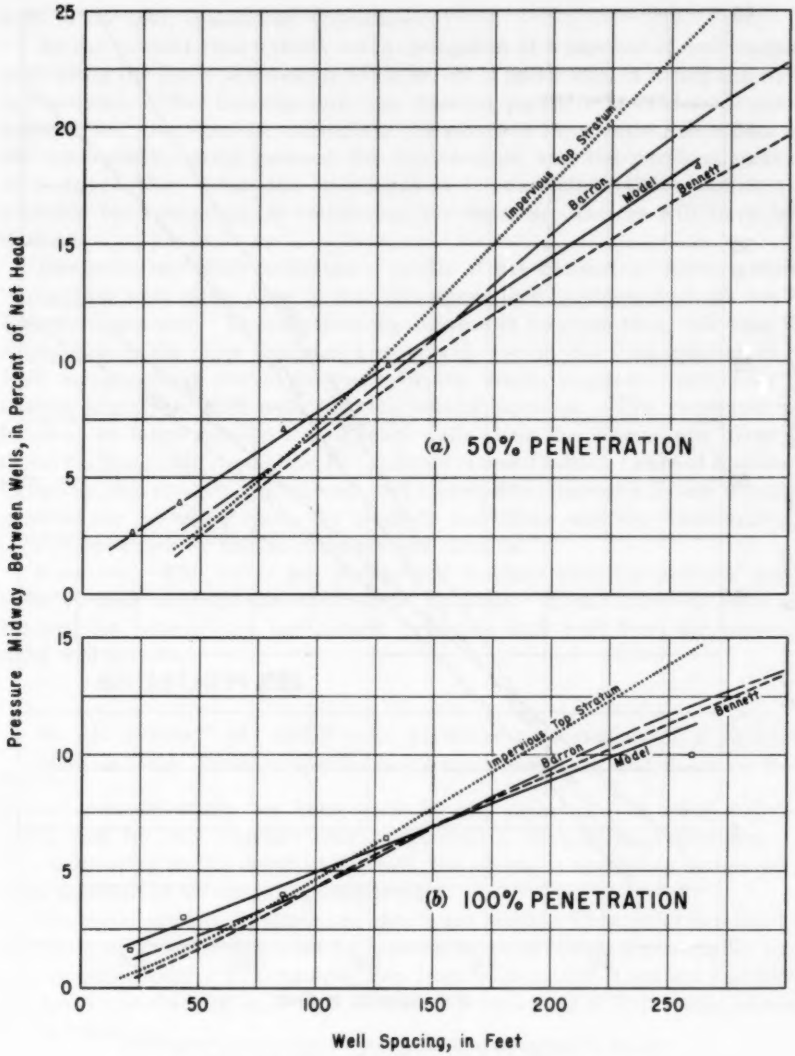

Fig. 23.—Comparison of Theoretical Pressures with Data from Model A-a-2 for Various Well Penetrations

would indicate results similar to those shown for the pressure midway between wells could also be made for the well flows.

Although Eqs. 1 and 2 make allowance for variation in the distance to the effective source of seepage, they do not permit a solution for a landside top stratum with a thickness or permeability different from the model prototype. Thus, the use of the approximate theoretical analyses seems preferable to the use of Eqs. 1 and 2. However, to utilize the theoretical analyses, an estimate

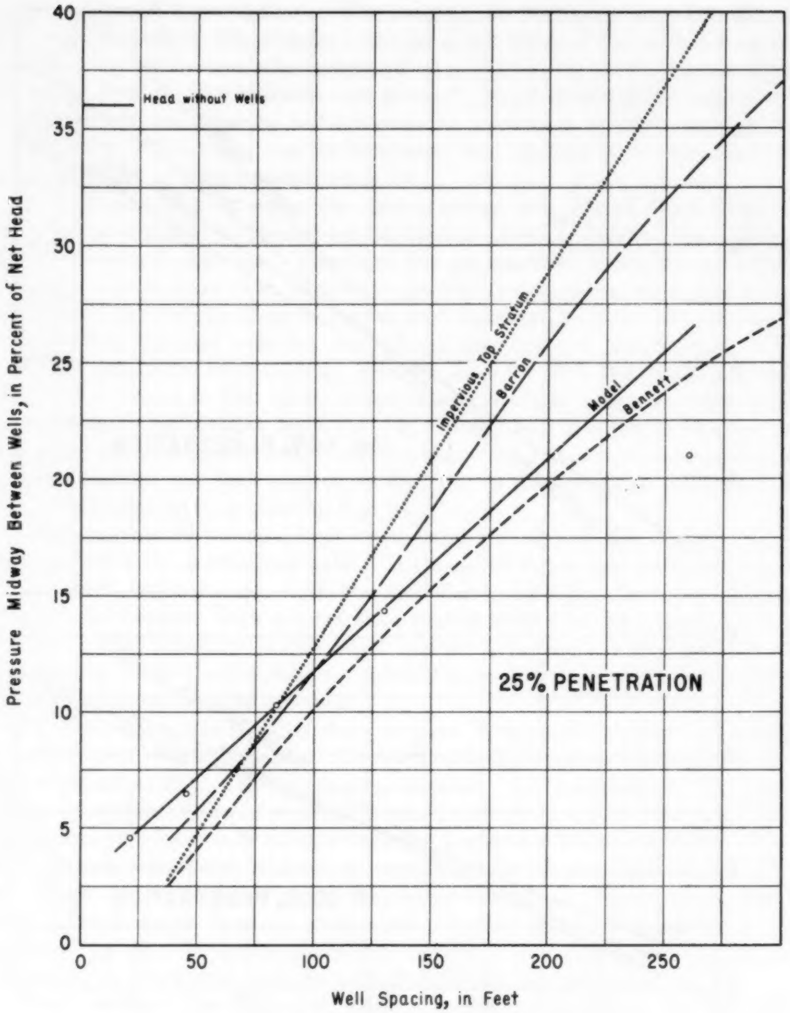

FIG. 24.—COMPARISON OF THEORETICAL PRESSURES WITH DATA FROM MODEL A-a-2 FOR 25% WELL PENETRATION

of the distance from the proposed line of wells to the effective exit for seepage is required. The best information would be from piezometers installed previously at the proposed site; but an approximation[7] can be made if the average top stratum thickness and the permeability are known. At the present time (1953) a precise determination of the effective top stratum thickness and the ratio of top stratum permeability to pervious stratum permeability, as they

[7] "The Effect of Blankets on Seepage Through Pervious Foundations," by P. T. Bennett, *Transactions*, ASCE, Vol. 111, 1946, p. 215.

exist in the field, is probably impossible.

At the present time (1953), an investigation of a number of underseepage sites along the lower Mississippi River levees is under way in which the writer has assisted. This investigation was directed partially toward an improvement in the procedure for estimating the effective top stratum thickness and the permeability ratio between the top stratum and the pervious stratum. It is hoped that, when this investigation is completed (1954), satisfactory methods for computing or estimating the necessary factors will have been derived.

One point on which preliminary results of the Mississippi River levee investigation were quite clear is that the minor local geologic features are extremely important. In fact, their significance is so great that, following the completion of the office design of a relief-well system, the final location of the wells along a levee should be made by the design engineer—modifying the spacing slightly to shift wells to more critical locations. This procedure was followed on large installations of relief wells along the Mississippi River between St. Louis, Mo., and Gale, Ill., in the St. Louis District, Corps of Engineers. In reality, this procedure illustrates that a completely precise solution is neither possible nor necessary since the landside conditions and the distribution of pressure and flow on the landside are too variable.

Summary.—The writer has endeavored to show that the authors' model tests verified approximate theoretical methods. These methods were developed for determining substratum pressures and well flows for pressure-relief well systems.

W. H. Jervis,[14] M. ASCE.—An excellent presentation of a thorough

[14] Resident Partner, Porter-Urquhart, Skidmore, Owings & Merrill, Associated, Casablanca, French Morocco.

hydraulic-model study has been made by the authors. The relief well is a useful tool for any engineer confronted with a deep-seepage problem. In fact, depending on the depth of the well, the saving in cost when compared to other methods of treatment is considerable.

The model studies illustrate one significant point. That point is related to quantities of water which must be handled so as to obtain a reasonably complete pressure relief. For example, flow from wells on 100-ft centers and 100% penetration as observed in Models B, C, and D for a head of 20 ft would amount to the following:

Model	Well Flow in gallons per minute	Well Flow in cubic feet per second
B–a	140	0.31
B–b	175	0.39
B–c	210	0.47
C	270	0.60
D	170	0.38

A head of 20 ft is possible on the main levees on the lower Mississippi River and a well spacing of 100 ft and penetration of 100% might be required

for adequate pressure relief. In the previously tabulated models for the given well spacing and penetration, pressure relief of from 85% to 95% was obtained. This relief is probably more than would be sought in the field. However, when the variations in the pervious layers underlying the levee are considered, it is doubtful that it would be possible to design a system which would maintain a definite percentage relief. Such a system would require valves (controlled by piezometer installations) placed on the wells.

When it is realized that the levees along the lower Mississippi River protect flat areas where drainage is naturally poor, it can be seen that for a large-scale installation the problem of water removal could, in some cases, be material. For example, there are several hundred miles of levee surrounding the delta area of the Yazoo River (Mississippi) and Mississippi River. At a rate of seepage of 0.4 cu ft per sec per 100-ft station, each mile of levee would yield 21 cu ft per sec of water which in some cases might have to be pumped back over the levee. This factor alone has led to some opposition to the installation of relief-well systems on the Mississippi River levees. Data from field installations of wells and piezometers are being gathered, and the progress of this work will be watched attentively by all interested persons.

The ultimate question is whether or not relief wells are necessary—that is, whether sand boils cause levee failures. The writer is convinced that they are a potential cause of levee failure and probably have been a cause in the past. However, determining the cause of a levee failure on a system as large as that on the Mississippi River is extremely difficult. It is not similar to watching seepage develop at a dam in a relatively confined space. If a failure occurs, it may occur suddenly, in a remote place, where there are no competent witnesses. The writer has questioned many people who have witnessed crevasses, and the causes as reported by them have varied from sand boils to "burrowing animals" to overtopping for the same crevasse. Obviously, such testimony is not convincing, and it is doubted whether any cause can be definitely assigned to any given crevasse. When a crevasse occurs, it is impossible to examine it afterward because there is nothing left but a big hole. The writer, nevertheless, is suspicious of underseepage as a leading cause of failures.

Meanwhile, the responsible engineers are confronted by a difficult problem. It is a problem with which the writer is familiar and with which he is sympathetic. The model studies described by Messrs. Turnbull and Mansur are a welcomed addition to this field of endeavor.

W. J. Turnbull,[15] M. ASCE, and C. I. Mansur,[16] J. M. ASCE.—The development by Mr. Bennett of generalized formulas for the design of relief-well systems from additional analyses of data from model B and electrical-analogy studies[5] of relief wells has added materially to this paper. The transformation of a stratified foundation to an equivalent homogeneous system

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[16] Chf., Design and Analytical Sect., Embankment and Foundation Branch, Soils Div., Waterways Experiment Station, Corps of Engrs., U. S. Dept. of the Army, Vicksburg, Miss.

[5] "Relief Wells for Dams and Levees," by T. A. Middlebrooks and W. H. Jervis, *Transactions*, ASCE, Vol. 112, 1947, p. 1321.

and comparison of the computed values of $P_a/(A\ S)$ with those obtained in model B is also enlightening.

Mr. Bennett has stated that the value of C in Eq. 3 is an unknown constant. In actuality, when it is assumed that the flow through the entire depth of the foundation is proportional to the gradient at the top of the sand (as Mr. Bennett assumes), then the flow,

$$Q_w = k_h\ D\ A\ (S_r - S_l) = C\ A\ (S_r - S_l) \quad \text{(12)}$$

thus,

$$C = k_h\ D \quad \text{(13)}$$

in which k_h is the average horizontal permeability of the pervious foundation. In model B, $C = k_h\ D = 0.10 \times 100 = 10$ cu ft per min per ft or 75 gal per min per ft which corresponds closely with the average value for C of 69 gal per min per ft for all the C-values in Table 2.

It has been implied by Mr. Bennett that the flow per unit length of levee from a line of wells is reasonably independent of the well spacing and penetration. The model data show that well spacing and penetration do have an effect on well flow; however, this does not invalidate Eqs. 3 and 5 as changes in S_l and P_a with different well spacings and penetrations are compensated by a corresponding change in Q_w.

In designing a well system for a given site by the use of the formulas (Eqs. 5, 8, and 9) offered by Mr. Bennett, it is necessary to determine first the permissible head beneath the top stratum which can exist at the landside toe of the levee without causing dangerous sand boils or piping. The well system is then designed so that the pressure midway between wells, P_m, does not exceed this allowable head. It is probably preferable to select first the desired well radius and the percentage penetration of the aquifer. With the distance to the effective source of seepage and the total design head, P_t, on the dam or levee known, a trial value for the well spacing can be assumed, and Eqs. 8, 9, and 5 (substituting $k\ D$ for C) can be solved simultaneously for Q_w, P_a, and P_m. This procedure can then be repeated for various values of well spacing until the well spacing having a head midway between the wells equal to the allowable head beneath the top stratum is obtained. In practical design problems, the hydraulic head losses in the well must also be considered as these losses add to the head midway between wells. It is also necessary to allow for end effects when the well system consists of a comparatively short line of wells, rather than an infinite line of wells, because the head midway between the wells near the ends of the system is greater than that at the central part of the system. Equal heads midway between wells on the ends of a short line of wells can be obtained by either decreasing the well spacing or by increasing the penetration of the wells at the ends of the well line.

It should be noted that in Table 3 the theoretical values of $P_a/(A\ S)$ for 100% penetration have been compared to model values of $P_m/(A\ S)$. Therefore, in order to obtain a comparison of Mr. Bennett's formulas with the model data, it is necessary to compute the theoretical values of $P_m/(A\ S)$. The computed values of $P_m/(A\ S)$ for the cases shown can be obtained by adding 0.11 to the underlined values in the column for 100% well penetration. All the other values shown in Table 3, both theoretical and observed, are $P_a/(A\ S)$.

Mr. Focht's comparisons of the model data and the heads computed from theoretical formulas are most interesting. As Mr. Focht notes, there is some slight scattering in the model data which results in a minor deviation from the theoretical values. However, the deviation is not considered to be important because, in the design of relief-well systems, many of the factors affecting the design cannot be evaluated with precise accuracy. It is therefore believed that the deviations between the model data and the theoretical values are of a minor nature in comparison with the uncertainties which can exist in the evaluation of factors in the field that affect the design of well systems.

Mr. Jervis has stated that the installation of relief wells will greatly increase the amount of seepage or water landward of a levee during periods of high water. The primary purpose of relief wells is to reduce the hydrostatic pressure on the landside (or downstream side) of levees or dams. By so doing, the hydraulic gradient from the river or reservoir toward the landside is increased, which in turn increases the landward flow to some degree. It is believed that this increase does not assume the proportions suggested by Mr. Jervis for the following reasons:

First, along most levee systems—and certainly along those in the upper and lower Mississippi River valleys—only a relatively small proportion (20% to 40%) of the levees is critical with respect to dangerous substratum pressures. Thus, the need for pressure relief is not continuous throughout the entire length of the levee system.

Second, the flow (resulting from surface runoff and general seepage during flood time) that must be accommodated by any existing surface drainage system landward of the levees is appreciable without relief wells. The increase in this flow which would be caused by the installation of relief wells along certain reaches of the levee would be relatively small during high water when one considers the total length of the levee system.

Third, most areas landward of levees that are critical with respect to underseepage are usually subject to heavy seepage or sand boils during periods of high flood stages without pressure-relief measures. If relief wells are installed, they will intercept a large part of this natural seepage without excessively increasing the total flow passing beneath the levee, particularly at the higher river stages. This is illustrated by subsequent examples of model data, field observations, and hypothetical computations.

In Model A-a-2 with a net head of 25 ft (Figs. 3 and 12)—a model fairly representative of conditions along the Mississippi River levees—relief wells on 100-ft centers penetrating half the depth of the pervious foundation reduced the hydrostatic pressure landward of the levee to 2.0 ft or only 8% of the net head on the levee; decreased uncontrolled seepage by 77%; and increased the total flow passing beneath the levee by 28%.

This example is supported by actual field measurements of seepage and substratum pressures along a levee reach without any wells (at Trotters, Mississippi) during the high water in 1950 and similar measurements made during a high water in 1951 of seepage and flow from relief wells subsequently installed at the same site. The total natural seepage in 1950 passing beneath

the levee, determined from piezometer readings and known foundation characteristics, was 156 gal per min per 100 ft of levee. In 1951, with a line of drainage wells on 100-ft centers along the toe of the levee, with 50% penetration of the sand aquifer and a net head on the levee of 7.55 ft, the flow from the wells was 123 gal per min per 100-ft station. Adjusted to the same head on the levee that existed in 1950 when the natural seepage was measured without wells, the well flow would have been 218 gal per min per 100-ft station. That is, at this site, the relief wells increased the total flow passing beneath the levee by 40%. There was no measurable natural seepage landward of the wells in 1951, primarily because the wells discharged into a collector ditch and a natural landward drainage ditch which carried the well flow without overflowing.

Another illustration of the amount that relief wells increase flow passing beneath a levee during high flood stages is demonstrated by a subsequent computation for a soil condition that could be critical in flood time. The maximum hydrostatic pressure that can exist landward of a levee with a height of 25 ft, a top stratum thickness of 8 ft, an assumed critical gradient of 0.8, and a saturated foundation is 6.4 ft. (Any substratum pressure greater than this would either raise the landside blanket or be self-relieved through sand boils.) Therefore, the net hydrostatic head causing landward flow would be $\Delta H = 25 - 6.4 = 18.6$ ft. The flow resulting from this decrease in head will emerge somewhere landward of the levee as general seepage or as flow from sand boils. If a sufficient number of drainage wells were installed so as to lower the residual pressure on the landside to approximately 2.0 ft, the landward flow would be that corresponding to a head drop of 23 ft (25 ft − 2 ft). That is, the wells would increase the landward flow by 24%.

On the basis of the foregoing model data, field data, and theoretical computations, it can be seen that the additional flow for a continuous length of levee resulting from drainage wells at intervals would be relatively small (10% to 30%) in most instances when one considers the total flow that must normally be cared for by the surface drainage system. An additional amount of water of from 10% to 30% will probably not materially add to the difficulties which already exist. As a corollary, if pumping is necessary, it will be made necessary primarily by the natural seepage and rainfall which would exist with no relief wells.

During low floods which may not completely saturate the landward top stratum, the increase of total flow beneath a levee caused by relief wells may be somewhat more than that shown by the previous examples for relatively high (12 ft to 15 ft or greater) floods. The objection to well flow during low flood stages has been partly met in the design of an extensive system of relief wells for levees in the St. Louis District, Corps of Engineers, United States Department of the Army, by installing a standpipe (on top of each well) which would automatically overflow when the hydrostatic pressure at the well exceeds approximately one fourth the thickness of the top stratum. When the standpipes begin to overflow, they are to be removed so that the well system can function as designed.

Mr. Jervis expressed the hope that the model studies would be extended to field studies. Such studies of full-scale field installations of relief wells have

been made. Information has been obtained on the functioning of a well system at Trotters during high river stages in 1951 and 1952. Also, extensive installations of relief wells have been and are presently (1954) being installed along Mississippi River levees in the St. Louis District. The writers have been and are presently closely associated with the field studies at Trotters and the design of the relief wells in the St. Louis District.

At the time the original model data were analyzed, it was considered desirable to assemble in a single volume certain relief-well-design formulas for various geological and stratigraphical soil conditions. Clarification of symbols, terminology, and various phases of well design could be accomplished in such a volume. This was done in an appendix to a publication.[17] Since the time of this publication, however, additional data have been developed and in one or two instances certain corrections to formulas have been made. Although further development and improvement in relief-well design will take place, another compilation bringing the 1949 publication[17] up to date would be extremely useful to engineers relatively unfamiliar with well-design problems.

[17] "Relief Well Systems for Dams and Levees on Pervious Foundations; Model Investigations," *Technical Memorandum No. 3-304*, Waterways Experiment Station, Vicksburg, Miss., November, 1949.

DISCUSSION OF THE STRUCTURE OF INORGANIC SOIL PROCEEDINGS-SEPARATE NO. 315

A. W. SKEMPTON[1]—Concerning the effects of leaching on clays of marine origin, a subject to which the author devotes considerable attention, there is available an appreciable body of field data, and it may be of interest to bring some of this forward in the discussion. In relation to the undisturbed strength of these clays, when in a normally consolidated condition (using this term in the sense defined by Terzaghi), leaching produces a definite but comparatively small decrease in strength. If the shear strength of a clay is c at a depth in the sediment where the effective overburden pressure is p, then in a normally consolidated clay the ratio c/p remains sensibly constant at all depths; except, of course, where drying has occurred in the upper zones, and provided the clay is reasonably uniform as shown by approximately constant Atterberg Limits. There is abundant field evidence to confirm this statement, and from this evidence it is found that an almost linear relationship exists between the ratio (c/p) and the plasticity index; not only within a single sequence of clay beds but also for all the clays for which data has so far been published. A few of these results are plotted in Figure 1.[2]

Now the effect of leaching a marine clay is to reduce the plasticity index, and hence it may be expected that the shear strength also will be lowered in accordance with the line given in Fig. 1. From tests near Oslo it has recently been shown by Dr. L. Bjerrum[3] that this is actually the case in a clay which has been leached by varying degrees in the course of its geological history. If, for example, a clay is deposited and normally consolidated with a plasticity index of 50, without leaching, it will have a (c/p) ratio of about 0.3 (see Fig. 1). But if it is now subjected to leaching and its plasticity index is reduced to 25, then its strength under the same effective over-burden pressure will be reduced by 30 per cent, or thereabouts, owing to a decrease in its (c/p) ratio from 0.3 to 0.2. A reduction of the plasticity index by the amount quoted above

1. Imperial College, London.
2. References given in this Figure are: A. W. Skempton (1948 a) "A Deep Stratum of Post-Glacial Clay at Gosport" 2nd Int. Conf. Soil Mech. 1, 145. A. W. Skempton (1948 b) "Vane Tests in the Alluvial Plain of the River Forth near Grangemouth" Geotechnique 1. 111. J. Brinch Hansen (1950) "Vane Tests in Norwegian Quick Clay" Geotech. 2. 58. A. W. Skempton & D. J. Henkel (1953) "The Post-Glacial Clays of the Thames Estuary at Tilbury and Shelhaven" 3rd Int. Conf. Soil Mech. 1. 302. F. P. Silva (1953) "Shearing Strength of a Soft Clay Deposit near Rio de Janiero" Geotechnique 3. 300.
3. Private communication.

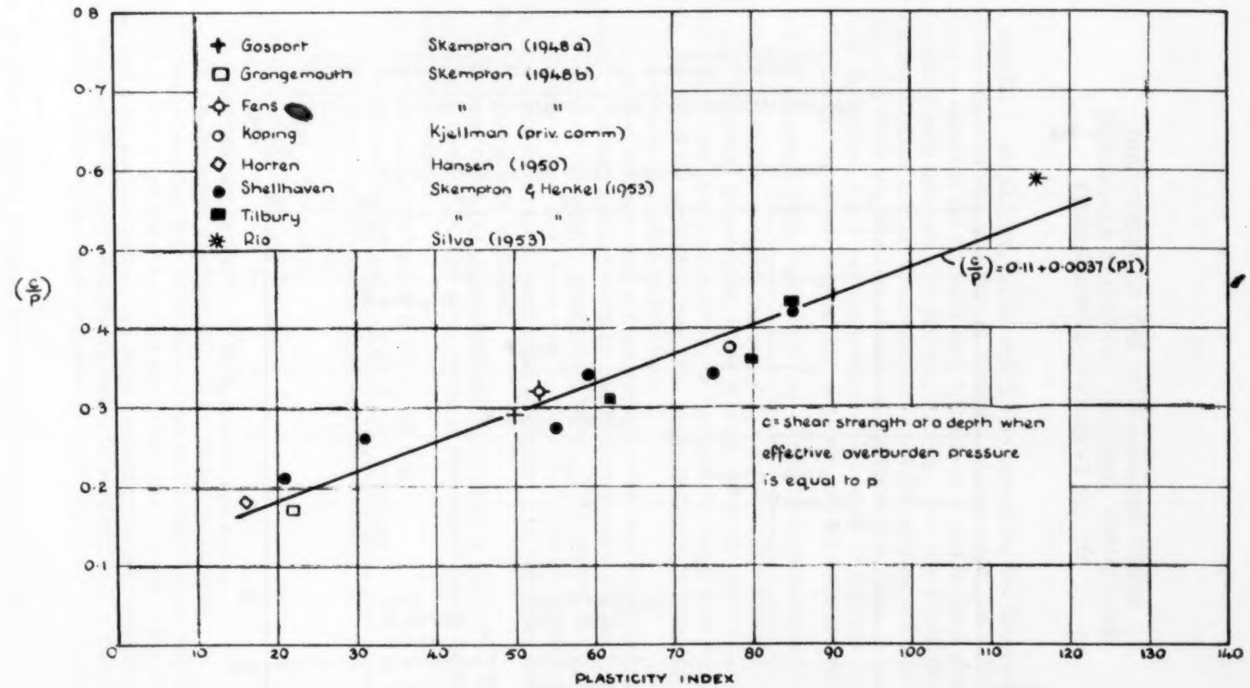
corresponds, however, to a very considerable degree of leaching. And as the undisturbed strength falls by only 30 per cent it may be said that leaching has no great effect on the undisturbed strength. This has also been demonstrated in the laboratory.

In contrast, the effect of leaching on the remoulded strength, and hence on the sensitivity, is very great. Marine clays which have not been leached appear to have a sensitivity of the order 2 to 4; and this may be due entirely or almost entirely to thixotropy. If, on the other hand, the concentration of salt in the pore water is reduced to about one-half of its original (sea-water) value, the sensitivity is increased to about 10; which implies a decrease in remoulded strength to roughly one-third of its original, unleached, value. This conclusion is drawn from the graph in Figure 2 giving all the field data known to the writer, for the relation between sensitivity and salt concentration in the pore water of clays of marine origin.[4]

The foregoing field evidence therefore indicates, firstly, that the undisturbed strengths of clays is slightly reduced by leaching, but, secondly, that the remoulded strength is greatly reduced by leaching. These facts seem to be in general agreement with the general theoretical views described by the author, and with the laboratory tests summarized in Table 2. In connection with Table 2 it would be interesting to know the Atterberg Limits of the clay before leaching and in the three tests quoted, and also the corresponding salt concentrations in the pore water.

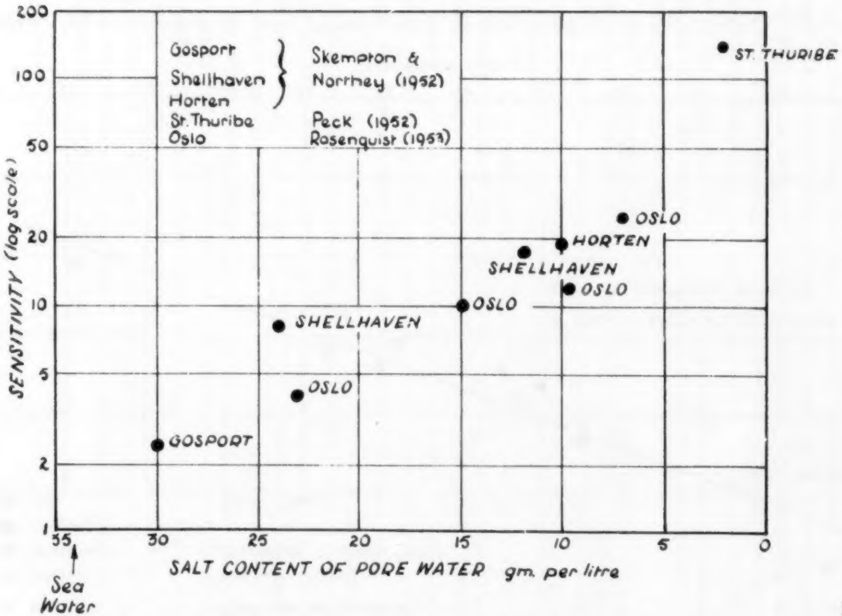
Since even the most drastic leaching to which the clay described in Table 2 was submitted reduced the undisturbed strength by only 50 per cent, and since in the field a more typical reduction seems to be about 20 to 30 per cent or less, and since, moreover, experiments (Skempton and Northey loc. cit.) have shown that leaching causes little chance in volume in an undisturbed clay, even when the clay is under load, it seems that once a clay has been deposited and consolidated, its undisturbed strength and compressibility are, to a first approximation, independent of any subsequent changes in salt concentration in the pore water. This suggests that the particles in a clay of marine origin must be effectively in contact with each other, when in the undisturbed state, and consequently calculations which have been published from time to time showing relatively great particle separations in clays, although possibly of some validity for the remoulded state, presumably are not applicable to marine clays in their natural state.

4. References given in Figure 2 are: A. W. Skempton and R. D. Northey (1952) "The Sensitivity of Clays" Geotechnique 3. 30. R. B. Peck (1952) Correspondence. Geotechnique 3. 142. I. T. Rosenquist (1953) "Considerations on the Sensitivity of Norwegian Quick Clays" Geotechnique 3. 195.



RELATION BETWEEN (c/p) AND PLASTICITY INDEX IN NORMALLY CONSOLIDATED SILTS AND CLAYS

Fig. 1



RELATION BETWEEN SENSITIVITY AND SALT CONCENTRATION

IN THE PORE WATER OF CLAYS OF MARINE ORIGIN

Fig. 2

DISCUSSION OF THE PILE LOADING TESTS, MORGANZA FLOODWAY CONTROL STRUCTURE PROCEEDINGS-SEPARATE NO. 324

JAMES F. McNULTY,[1] A.M. ASCE.—In computing the theoretical bearing capacity of a pile driven through clay into sand, the authors utilize an equation for ultimate skin friction on a pile in sand which is unjustified both theoretically and empirically. In a two-dimensional problem of plane strain, Coulomb's equation that sliding will occur in an ideal sand when the shearing stress on any surface is equal to the product of the normal force and the tangent of the angle of internal friction ($s = \sigma \tan \phi$); the three-dimensional problem has not been solved. The authors' equation, thus, that $f_s = \gamma \left(D_f - \frac{h}{2}\right) \tan \phi$ is theoretically unwarranted. In similar manner, the use of $f_s = c$ for a clay is unjustified; its use, however, is prevalent since it often, as in the case reported, checks test data fairly accurately. If the authors are proposing the use of $f_s = \sigma \tan \phi$ for empirical reasons, more evidence is required since its influence is negligible in this case.

The writer believes that a somewhat sounder theoretical solution for ultimate bearing capacity could be obtained by choosing ϕ and f_s (sand) on the basis of the sand's penetration value. This results in approximate values of $f_s = 2000$ p.s.f. and $\phi = 35^o$. Incidentally, it appears that the small additional cost involved to determine ϕ by laboratory tests would have been well expended.

Table I indicates the percentage error involved in computing the theoretical capacity for the piles tested in this report based on the following assumptions:

(1) Imbedment of "a" piles in clay = 65 feet
(2) Imbedment of "b" and T piles in clay = 70 feet
(3) Imbedment of "b" and T piles in sand = 5 feet
(4) Surcharge = 10 feet
(5) f_s (clay) = 660 p.s.f.
(6) f_s (sand) = 2000 p.s.f.
(7) $\phi = 35^o$
(8) Point bearing in sand = $A\gamma D_f N_q$
(A = point area, γ = 50 p.c.f., other terms negligible)

It should be noted that while the errors are within estimating accuracy, it is due to the accuracy with which the skin-friction value of the clay has been approximated. As table I reveals, the percentage error for the piles in the clay stratum is under 10 percent; this is exceptional accuracy. Table II has been prepared to show the error associated with

1. Structural Engr., National Advisory Comm. for Aeronautics, Langley Field, Va.

TABLE I

Pile No.	Pile type	Theoretical capacity (tons)	Failure load by gross curve (tons)	Percentage error
c-1-a		130	125	4
c-1-b	24-in. diam.	438	320	37
T-1		170	130	31
c-2-a	8-in. tip,	72	78	8
c-2-b	tapered	117	145	19
T-2		87	120	27
c-3-a	12-in. diam.	65	56	14
c-3-b		152	145	5
T-3		85	85	0
c-4-a	18-in. diam.	97	80	21
c-4-b		278	230	21
T-4		128	125	2
c-5-a	24-in. diam.	130	137	5
c-5-b		438	305	44
T-5		170	>160	--
c-6-a	30-in. diam.	163	165	1
c-6-b		633	450	40
T-6		213	>150	--
c-7-a	22-in. square	152	135	13
c-7-b		487	285	70

Average percentage error - "a" piles = 9
"b" piles = 34
T piles = 15
all piles = 20

TABLE II

Pile No.	Pile type	Friction from sand stratum included: Theoretical tip capacity (tons)	Tip failure load (tons)	Error percent
c-1-b	24-in. diameter	298	185	+ 61
c-2-b	8-in. tip, tapered	40	61	- 34
c-3-b	12-in. diameter	82	85	- 4
c-4-b	18-in. diameter	173	144	+ 20
c-5-b	24-in. diameter	298	157	+ 90
c-6-b	30-in. diameter	458	272	+ 68
c-7-b	22-in. square	323	140	+131

estimating the pile's capacity in the sand stratum; the following assumptions are made:

(1) Tip load failure = "b" load failure - 70/65 "a" load failure

(2) Theoretical tip failure = $A\gamma D_f N_q + 2\pi R h f_s$

It is apparent that the estimates obtained are unreliable; analysis, however, does indicate a worthwhile particular. The tabulation reveals that the theory gives accurate results at a pile diameter of 12 inches, underestimates at diameters less than 12 inches, and overestimates at diameters greater than 12 inches; in other words, the unit failure point-bearing load is inversely affected by point area.

Assuming f_s (sand) = 2000 p.s.f. and subtracting its influence from capacities in table II results in the following tabulation:

TABLE III

Pile No.	Pile type	Theoretical tip capacity (tons)	Tip failure load (tons)	Error percent
c-1-b	24-in. diameter	268	155	+ 73
c-2-b	8-in. tip, tapered	30	51	- 39
c-3-b	12-in. diameter	67	70	- 4
c-4-b	18-in. diameter	150	121	+ 24
c-5-b	24-in. diameter	268	127	+111
c-6-b	30-in. diameter	420	234	+ 80
c-7-b	22-in. square	286	103	+180

Figure 1 illustrates how the test failure loads fall below the theoretical failure loads for the larger diameter piles. It is realized that insufficient test points are available to plot the test curve properly; however, there is an indication that Terzaghi's equation for shallow footings could be utilized with a tip area factor (α) for estimating the point capacity of a pile.

A formula for the tip capacity of long piles would then be of the form

$$Q_p = \alpha A \gamma D_f N_q$$

Values of α as functions of tip area could then be plotted. For example, figure 2 is based on the curves of figure 1. The equation for the straight portion of the curve is

$$\alpha = 2.2 - 0.6 \log 144A$$

Thus, the complete equation for estimating the capacity of a "b" type pile would be

$$Q = 2\pi R(l f_s + h f_s') + (2.2 - 0.6 \log 144A) A \gamma D_f N_q$$

l imbedment in clay
f_s skin friction in clay
h imbedment in sand
f_s' skin friction in sand
Units are feet and tons.

There is, at present, no satisfactory theoretical method available to estimate either the skin-friction value in sand or point bearing capacity. It is unknown whether the refinement of α suggested by the writer is

empirically sound. Hindering progress greatly is the paucity of joint soil and test results such as the authors' report. The authors are to be congratulated both for their concise, complete report and for the methods utilized in the testing. This is exactly the type of paper which is urgently needed; it should be used as a pattern for future testing and reporting. Figures 1 and 2 appear on pages 27 and 28.

JOHN W. DUNHAM,[1] M. ASCE.—The sort of information contained in this paper is the stuff of which a science of pile design must be built. The data on piles C-4-a, C-4-b and T-4 is particularly valuable in this respect because of the detailed quantitative data on the piles, the soil and the tests.

The question of what constitutes failure of a pile under test loading is important. A confusing ambiguity would be resolved if engineers can agree on a definition.

The writer believes that failure should be defined in terms of the movement of the pile head, since that is the quantity that effects the behavior of the superstructure. In his own work he has used the following criterion to determine from test results whether or not a pile is satisfactory for the design load, "For the test to be considered successful, the head of the pile must not settle more than 0.01 inches per ton under any increment of test load before the test load equals the design load, nor more than 0.02 inches per ton under any increment of load before the test load equals twice the design load." That is, among other things, a test pile must carry at least twice its design load before the slope of the load-deflection curve becomes steeper than 0.02" per ton. That slope, then, is a criterion of failure. The slope of the load-deflection curve for pile C-4-a exceeds this amount at 90 tons. That for pile C-4-b exceeds it at 260 tons.

The writer concurs in the authors' concern that the transient supporting value of the clay be discounted and feels that the design selected to carry the structure was proper. While he does not question the propriety of the design, he does wish to question certain interpretations of the data that are of importance to the understanding of pile behavior.

The values of the angle of internal friction that the authors have derived from the test results appear to be very low. The resistance to the driving of the sample spoon, the pile driving record and the 80% relative density cited, all indicate a dense sand. It is further described as "a uniformly graded fine sand composed of subrounded to subangular grains." The representative values of ϕ given in "Soil Mechanics in Engineering Practice" by Terzaghi and Peck indicate that the angle of internal friction for such a sand should be in the neighborhood of 40°.

The load-deflection diagram for pile C-4-a shows the typical failure of a pile in clay. The total settlement under the last loading was about 3.13 inches or .035" per ton of load. The settlement was probably limited by the available travel of the jack. It would be valuable to know what the load was on the jack when the pile head stopped moving. This would make it possible to estimate the value of sliding friction through

1. Superv. Structural Engr., Public Bldgs. Service, Washington, D.C.

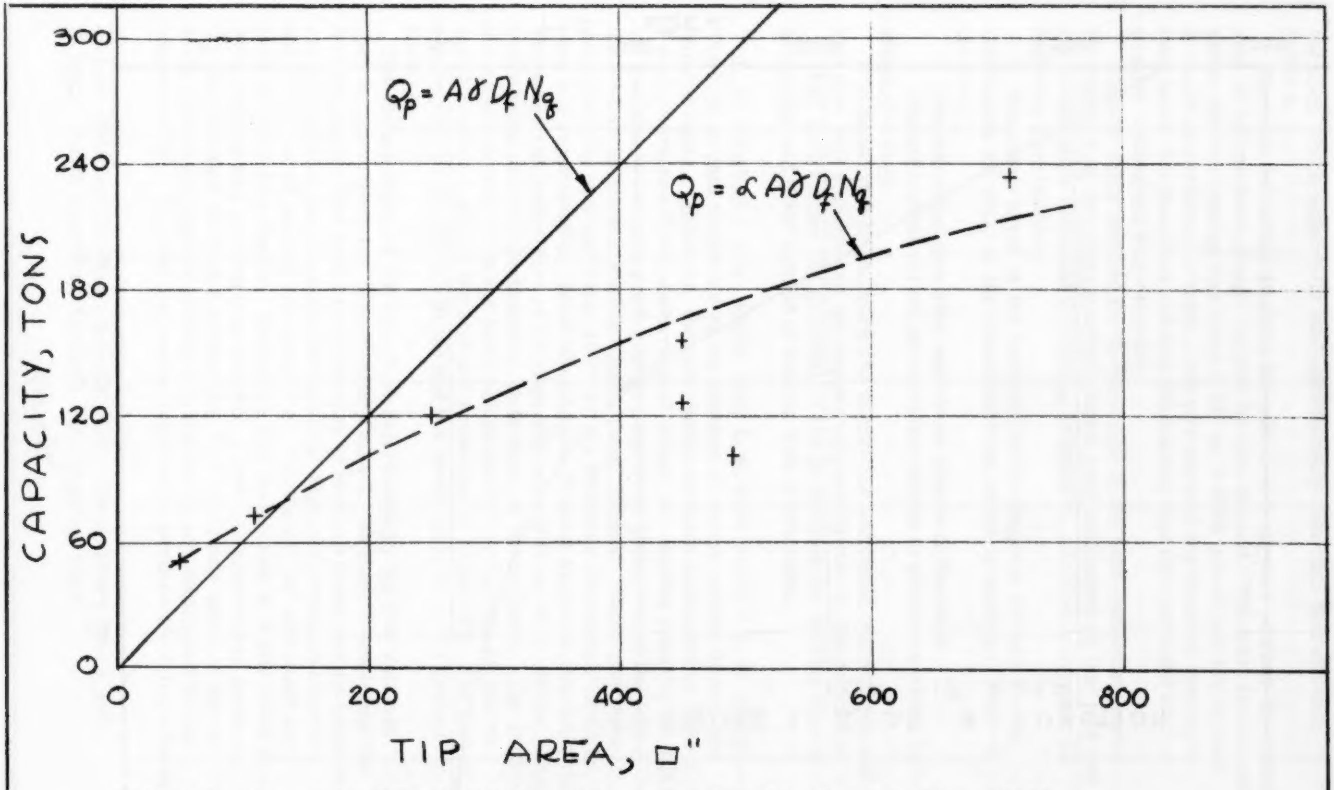


FIGURE 1: CORRELATION OF THEORY & TEST DATA

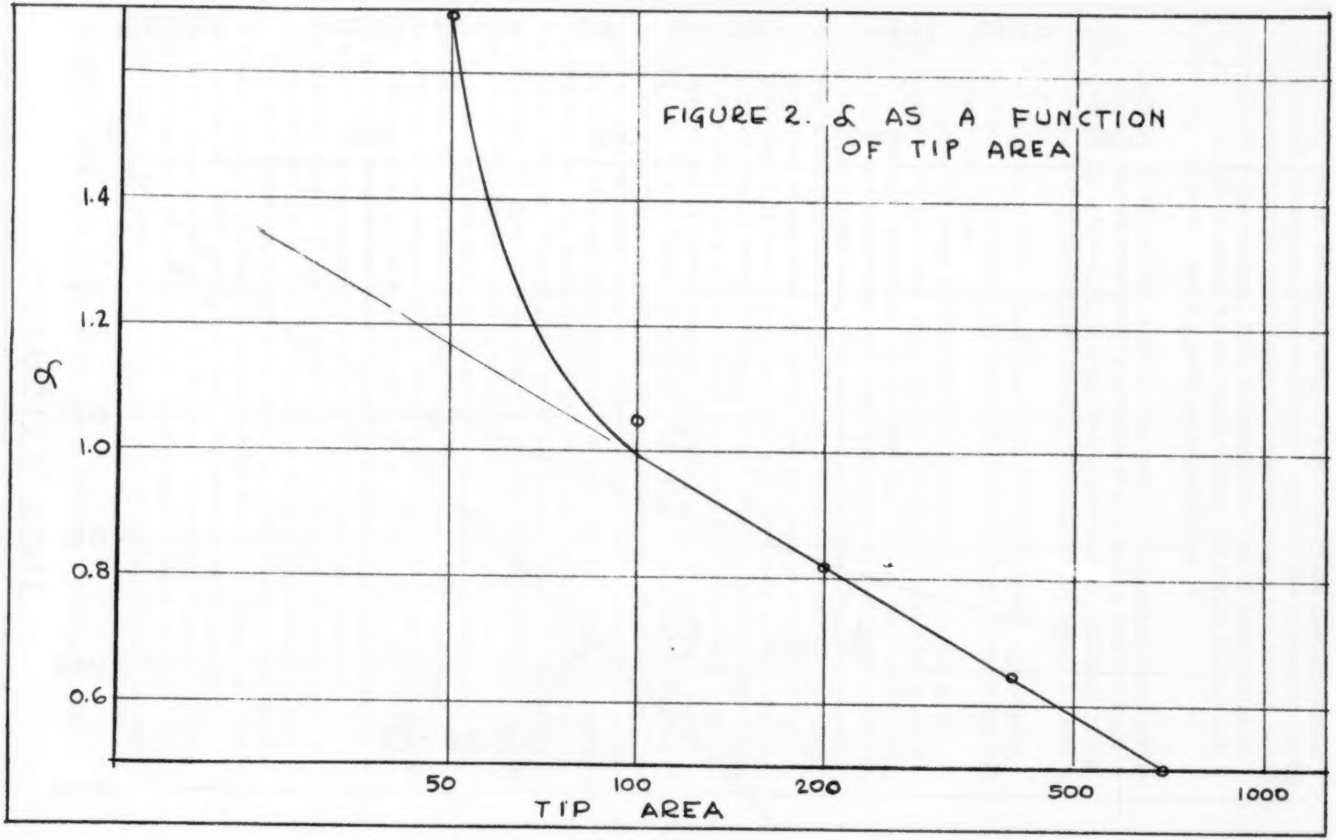


FIGURE 2. δ AS A FUNCTION OF TIP AREA

the clay, which apparently was substantially less than that of static friction.

The load-deflection diagram for pile C-4-b appears to show the same sort of failure even though the pile tip was in sand. The total settlement under the last loading was about 2.46" or 0.0095 inches per ton of load. It would be pertinent to know if the pile showed any signs of coming to rest under the final load and why the loading was not continued until a greater settlement was recorded.

The low value derived for ϕ, the shape of the load-deflection curve for pile C-4-b, the comparatively small total settlement of that pile and the lower value indicated for sliding friction on pile C-4-a suggest the following possibilities:

1. The sharp break in the load-deflection diagram for pile C-4-b represents the start of the transition from static to sliding friction in the clay.
2. The transition preceded any shear failure of the sand.
3. The settlement did not proceed far enough to produce a shear failure in the sand.

It is hoped that the authors can supply the additional information and that they will comment on the suggested possibilities in their closure.

They have earned the gratitude of the profession for making this information and their method of dealing with it available.

L. D. GRAVES,[1] A.M. ASCE, AND J. E. BINCKLEY,[2] J. M. ASCE.— The type of data presented by the authors is sorely needed in soil mechanics today. However, the calculation methods presented in Figure 8 would have predicted nowhere near either the bearing capacity or pull out resistance actually produced by a series of test piles at the site of the sewage treatment plant for South Bend, Indiana. These pile tests were made in May and June, 1953, by the Raymond Concrete Pile Company and supervised by Ralph Moorhouse, Project Engineer, for Consoer Townsend and Associates of Chicago with advice from W. L. Shilts, Head, Department of Civil Engineering, University of Notre Dame. The load-movement curves for these tests are presented in Figures 14 and 15 and the boring logs for the site are presented in Figure 16. The site is adjacent to the St. Joseph River and the water table is at the ground surface.

The failure loads of the test piles determined by the five methods used by the writers are presented in Table 7. In Table 8 are listed the friction angles that would be necessary to produce these failure loads as calculated from the Terzaghi Method A of Figure 8, using an effective soil weight of 50 pounds per cubic foot. Obviously, the soil could not have such friction angles. Reasoning that the driving of a pile could produce passive soil pressure on its side, a new calculation was made using the Terzaghi Method A except that "f_s" was made equal to the average Rankine's passive pressure in the sandy soil multiplied by the tangent of "ϕ". The friction angles required to produce the failure loads

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by this new calculation were reasonable as shown in Table 8.

Comparison of the boring logs of Figure 16 with the soil conditions at Morganza and a little computation shows immediately how the reasonable "ϕ" values of Table 3 were obtainable with Terzaghi Method A at the Morganza site but were not obtainable at South Bend. The shallow sand embedment at Morganza coupled with the 80 feet of clay overburden made the point bearing capacity of the pile a much larger proportion of the supporting power than the side friction in the sand. Thus the "f_s" had little effect. On the other hand, the South Bend site had only a little soft peaty overburden with most of the pile embedded in a sandy soil. This caused the side friction to be by far the larger portion of the supporting power of the pile and of course the whole of its resistance to being lifted. Thus "f_s" became the most important factor and could consequently be evaluated more accurately.

On the basis of the results of the South Bend tests, the second of the three conclusions should be revised to read: a reasonable estimate of the bearing capacity or pull out resistance of a pile driven into sand may be made by using Method A or D using the angle "ϕ" appropriate to that sand and assuming that the lateral pressure against the pile is Rankine's passive pressure for the sand.

CHARLES I. MANSUR,[1] M., ASCE, AND JOHN A. FOCHT, JR.,[2] A.M., ASCE.—As Mr. Dunham emphasized, the movement of the pile head is important in the interpretation of pile loading tests and should be given consideration in the design of a structure and the pile foundation. However, the question of what constitutes failure of a pile can be answered only in light of the proposed structure and its desired characteristics. Usually a considerable amount of movement of the pile head will result from elastic deformation of the pile and the surrounding soil as the load is applied. Generally, this type of movement is not detrimental to a structure, in view of the fact that it occurs as the weight of the structure is progressively increased during construction. Therefore, it is important to distinguish between complete failure of a pile and whether or not the movement of the pile head is more or less than an arbitrarily selected amount. Although deformation of the pile head is important in the design of a structure and some minor damage might occur if it is excessive, knowing what the capacity of the pile is from a failure standpoint appears to be of much greater importance.

The criterion to determine a design load proposed by Mr. Dunham is considered to be satisfactory for relatively short and/or lightly loaded piles, and it was one of the other procedures initially used for determination of the failure load of a test pile but it was discarded after preliminary trials. For long piles heavily loaded, the elastic deformation of the pile and soil may be initially greater than 0.01 in. per ton of load as happened for piles C-1-a, C-2-a, C-3-a, C-3-b, and C-5-a. According to Mr. Dunham's criterion, these piles would not be suitable to

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TABLE 7

TEST PILE FAILURE LOADS
SOUTH BEND SITE

	Method of Failure Load Determination					
Pile	0.25-in. Net Movement Tons	Tangent Intersection Gross Curve Tons	Inspection of Gross Curve Tons	Slope of Net 4 times Slope of Elastic Tons	Inspection of Net Curve Tons	Average of Five Methods of Interpretations
A-2*	–	45.5	45.0	47.4	47.2	46.3
A-4*	25.2	17.8	15.0	16.5	18.5	18.6
B-2*	36.8	33.4	35.0	38.5	32.9	35.3
C-2-1**	–	33.0	33.0	–	45.0	37.0
C-2-6*	–	29.3	28.0	–	–	28.7
D-2*	48.5	26.5	29.5	27.0	30.0	32.3
D-3-2**	–	51.0	41.0	71.0	95.5	64.6
D-3-3*	49.8	41.5	36.9	47.0	43.7	43.8

*Tension Test
**Compression Test

TABLE 8

INDICATED INTERNAL FRICTION ANGLE OF SOIL
SOUTH BEND SITE

Pile	Size & Type	Pile Length Feet	Method of Analysis Terzaghi "A"	Modified "A"
			Indicated Ø Degrees	
A-2*	Raymond Step Taper 10"-14"	34.83	45	23
A-4*	Raymond Step Taper 10"-12"	18	61	30
B-2*	Wood 8"-12"	31	49	25
C-2-1**	Wood 8"-10"	21.83	51	34
C-2-6*	Wood 8"-10"	21.83	64	32
D-2*	Raymond Step Taper 10"-13"	25	54	27
D-3-2**	Raymond Step Taper 10"-13"	28	52	32
D-3-3*	Raymond Step Taper 10"-13"	28	56	28
Avg.			54	29

*Tension Test
**Compression Test

carry any load. Mr. Dunham states that the "slope (of the load deflection curve), then, is a criterion of failure," but in determining a value of 90 tons for pile C-4-a, and 260 tons for pile C-4-b, he apparently selected the point at which the total movement became more than 0.02 inch per ton of load. Averaging results indicated by several methods of analysis, if the test is not performed under a specific building code, appears to be most desirable procedure for determining the pile failure load.

The value of the angle of internal friction, Ø, from laboratory tests on undisturbed samples of similar sands in the lower Mississippi Valley is about 35°. No triaxial tests were performed on undisturbed or remolded sand samples from the site of the pile tests. The high driving resistance of the standard split spoon in the sand at this location is believed due in part to the depth of the sand. Extensive investigations of split spoon driving resistance and cone penetration tests by the Waterways Experiment Station[3] have indicated that the penetration resistance is materially affected by physical dimensions (depth, water table, etc.) and soil properties other than relative density and angle of internal friction. It is doubtful if the value of Ø is as high as 40° as suggested by Mr. Dunham, but it may be more than the 30° value indicated.

No measurements were made to determine the sliding friction in the

3. Waterways Experiment Station. "Standard Penetration Tests, Reid-Bedford Sand, Mississippi River." Potamology Investigation Report No. 5-5, May 1950.

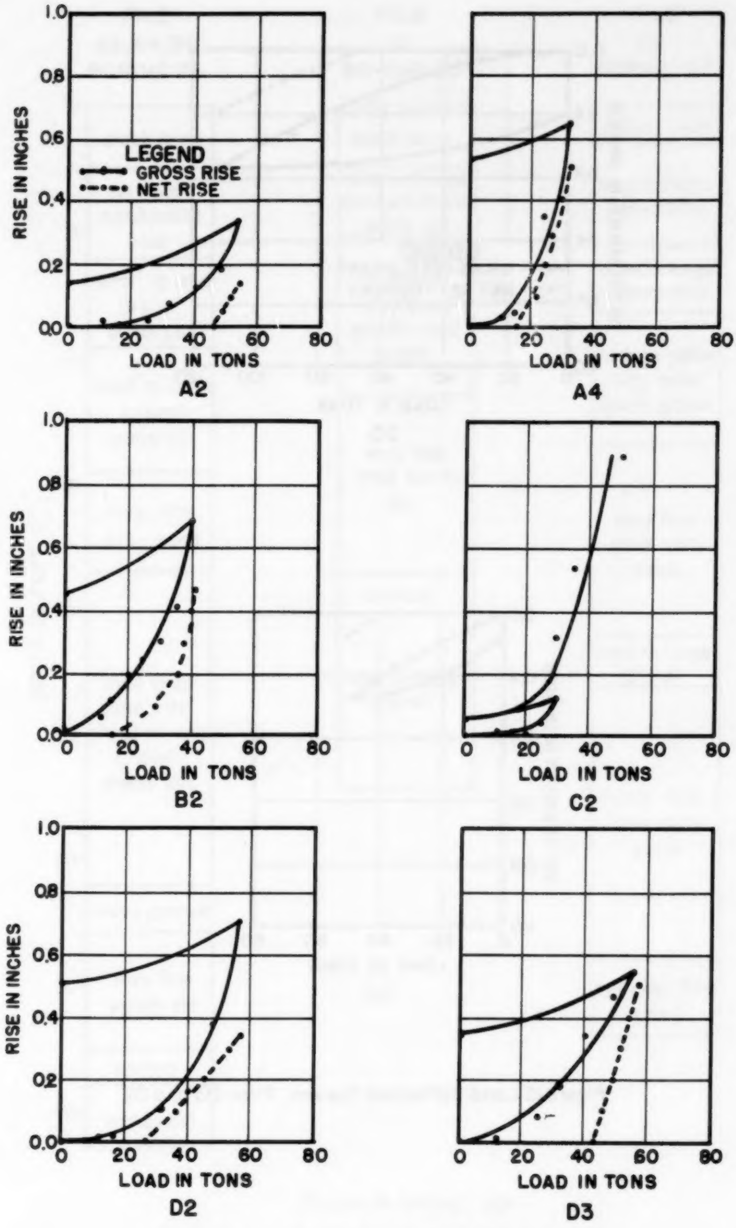
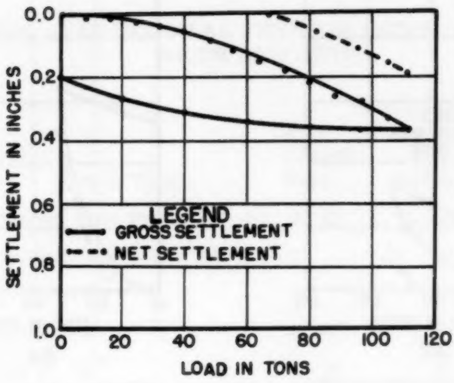
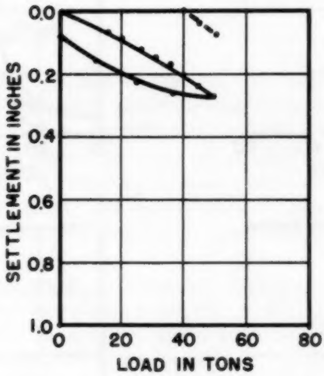


Figure 14· Load Rise Curves, Piles A2 through D3



D3



C2

Figure 15-Load Settlement Curves, Piles D3 and C2

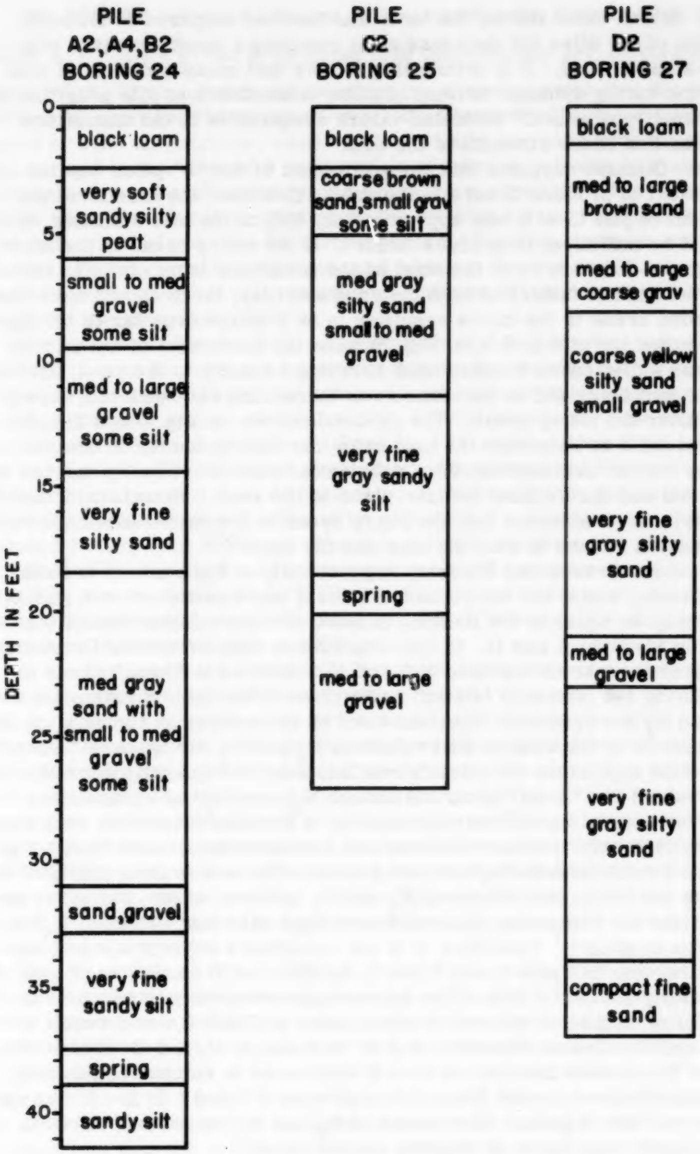


Figure 16 · Boring Logs

clay. It was noted during the tests that the load required to maintain motion of the piles did decrease after reaching a maximum (see pile C-5-a and C-6-a). It is interesting to note that measurements of skin friction during dynamic driving of piles in another test pile program in the same type of soil[4] indicated values comparable to the unconfined undisturbed shear strength of the soil.

Mr. Dunham suggests that the settlement of the "b" piles was not sufficient to produce failure in the sand. However, the maximum settlement of pile C-4-B was approximately 15% of the pile diameter, which should be sufficient to produce failure. If the sharp break in the load-deflection diagram were the start of the transition from static to sliding friction and not failure in both the sand and clay, there should have been a second break in the curve resulting in an S-shape (similar to the tip load curve for pile C-5-b on Fig. 6) when the movement of the pile tip became sufficient to develop load-carrying capacity in the sand; but the settlement continued to increase at an increasing rate with increasing load after the sharp break. The tip-load curves on Fig. 5 and Fig. 6 are intended to illustrate the load being carried by the tip of the pile. These curves indicate that very substantial loads were being carried by the sand and that failure did take place in the sand. Therefore, it is still the writers' conclusion that the sharp break in the load-settlement curve represents failure in both the clay and the sand.

Messrs. Graves and Binckley suggest that, on the basis of certain pile loading tests, the coefficient of lateral earth pressure on a pile in sand may be equal to the Rankine passive pressure rather than 1.0 as used in Methods A and D. In correspondence with Professor Graves it was learned that he had used N_q' and N_γ' values for "local" shear in computing the indicated friction angle given in his table 8 rather than N_q and N_γ for "general" shear as used by the authors in figure 8. It is the opinion of the writers that values of N_q and N_γ for "general" shear are more applicable for piles driven into sand and gravelly deposits than values for "local" shear. Although N_q' and N_γ' are sometimes used for computing the bearing capacity of footings founded in very loose sands, it is believed that vibration and displacement caused by driving a pile in a cohesionless deposit would density the soil in the vicinity of the pile to the extent that values of N_q and N_γ (general shear) are more appropriate for computing skin friction along a pile than values of N_q' and N_γ' (local shear). Therefore it is not considered appropriate to compare the data in Table 3 and Table 8, as different formulas were used in computing the data. Use of the Rankine passive pressure on a pile in computing angles of internal friction listed in Table 3 would result in even smaller indicated values of ϕ by Methods A, C, and D. The writers are of the opinion that the value of K to be used in computing the skin friction of piles founded in sand lies between 0.5 and 3.0; $K = 1.0$ as used by the writers together with values of N_q and N_γ for general shear is considered reasonable or slightly conservative.

4. American Railway Engineering Association. Advance Report of Comm. 8, "Steel and Timber Pile Tests, West Atchafalaya Floodway, N.O.T. &M., Railway," Bul. 489, p. 149, Sept-Oct 1950.

Mr. McNulty objects to the use of the equation for skin friction in sand, $f_s = \gamma \, (D_f - \frac{h}{2}) \tan \phi$, as being both theoretically and empirically unjustified. It was assumed in writing the above equation that the coefficient of lateral earth pressure was 1.0. Theoretically, the frictional force of sand on concrete, wood or steel should increase with increasing normal force. Use of ϕ as the angle of friction between soil and concrete is used routinely in earth pressure computations for retaining walls when consideration is given to the friction between the soil and the wall. Therefore, Mr. McNulty's general objection appears unwarranted. In any case, Mr. McNulty and Messrs. Graves and Binckley are correct in pointing out that the influence of skin friction in sand was small in the Morganza tests.

Nevertheless, Mr. McNulty cannot obtain a, "sounder theoretical solution . . . . on the basis of the sand's penetration value," as the correlation of ϕ and penetration resistance is not only empirical but one with a large possible error. The value of f_s = 2000 pounds per square foot apparently is from Terzaghi and Peck (p. 461) which is based on tests on short piles. Terzaghi and Peck state: "In every sand the average skin friction per unit area of contact . . . . increase with increasing depth." Thus, the skin friction to be expected at a considerable depth might be more than 2000 psf. The skin friction developed by sand at a depth of 85 feet using $\phi = 30^{\circ}$ and a lateral earth pressure equal to the Rankine passive pressure is 7350 psf. For an earth pressure coefficient of 1.0, the value of f_s would be 2450 psf, which is very little different from Mr. McNulty's value of 2000 psf.

In the development of Tables I, II, and III in an attempt to show the percentage error involved in estimating the pile capacity, Mr. McNulty did not give any consideration to the actual variation in penetration. Thereby, he included in his tabulated error the potential error which would be involved in predicting the penetration into sand. If a curve such as his Fig. 2 and the equation derived therefrom are to be developed, the actual penetration of each pile into sand should have been utilized. Furthermore, as the skin friction in the sand may have been somewhat greater than the value used by Mr. McNulty, the refinement suggested does not appear to be justified. It also appears that Mr. McNulty neglected to carry forward the algebraic sign of his computed percentage error in averaging the error.

Fig. 9, as well as Mr. McNulty's Table II, indicates that the potential error in estimating the bearing capacity of the pile til is large, and possibly unconservative for large diameter piles. The close check of Mr. McNulty's theoretical computation for a 12-inch pile and the test data is a function of the value of f_s and ϕ assumed in his computations. Fig. 9, which is based on the indicated value of $\phi = 30^{\circ}$, shows an underestimation by the theoretical analysis for piles less than 20 inches in diameter. The writers feel that the scatter of data is too wide to draw more than very general conclusions.

The writers disagree with Mr. McNulty that the use of the cohesion of clay for the skin friction on piles is unjustified, particularly in soft clays. Mr. McNulty states that $f_s = c$ often checks test data. This

empirical relationship is ample justification for its use.

The writers appreciate the interest shown in this paper and the contributions made by the discussers.

Correction to Proceedings-Separate 324—On page 324-11, in the fourth paragraph under the heading "Summary", the first sentence should read "***Method D, Jaky***".

DISCUSSION OF FOUNDATION EXPLORATION—DENVER COLISEUM PROCEEDINGS-SEPARATE NO. 326

ANTON TEDESKO,[1] M. ASCE.—It is the purpose of this discussion to present the structural background leading to the foundation problems solved by Dr. Peck.

After the donation of a site for the new Denver Coliseum, a multipurpose structure of 308' x 400' overall dimensions, it was determined that the surface of the natural ground at this location was about 15' higher than the level where it seemed best to place the arena floor. Having in mind the often told tale of a dome, poured directly on a mound of dirt which later was removed from under the dome, the designers visualized that it would take only a low height scaffolding to build a monolithic roof structure on the existing ground and that the excavation for a natural arena, subfloor and basement area, could be made afterwards. When advantage was thus taken of the natural contours and of the availability of the results of preliminary borings, indicating that good gravel would be encountered at a reasonable footing level, an arch type structure became the logical choice for the structural system, especially in view of the space and dimensional requirements. Reinforced concrete was obviously economical for a structure predominantly in compression where a sufficient number of similar construction units permitted effective re-use of movable form work. Several alternate preliminary designs were made and priced; a concrete arch-shell structure proved to be the soundest solution. Figure 1 shows a cross section indicating the span-rise relationship and the size of structural members. Figure 2 gives an overall view of the building during construction.

The arch ribs, with their axes curved to the form of a catenary, were supported on the heavy frames of a leanto structure. The great stiffness of these buttresses permitted the slender shaping of the arch. These buttresses have arched openings providing a continuous open promenade around the building. When all components of the structural system were considered, the arch ribs proved to be almost fully restrained. Decreasing the size of the buttresses and thereby permitting a lesser restraint at the springing line of the arch, would mean increasing the size and weight of the arch, thereby increasing its cost. Furthermore, decreasing footing movements by enlarging the foundations would mean additional foundation cost, which might be justified by obtaining greater rigidity of arch supports.

Knowing the soil pressure on the arch footings, and with the soil data obtained by Dr. Peck, settlement, translation and rotation of these footings were predicted. The predicted compression of the soil, 0.15" per

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1000 lbs. per sq. ft., was included in the calculations of the structural system. These calculations indicated the amount of extra steel required in the superstructure to take care of movements in the arch foundations and made it possible to judge whether it was more economical to provide for extra strength in the arch or for the construction of less yielding foundations. It was not necessary to investigate the use of piles or the provision of tension ties designed to prevent the spreading of the arch supports.

The resulting design for the Denver Coliseum shows an economical balance between the saving in the cost of the arch compared to the expense of more rigid foundations. Considerations of settlement governed the foundation design. It proved advantageous from a standpoint of overall economy to limit footing rotations to an equivalent of 0.6" differential settlement between toe and heel. The maximum measured differential dead load settlement between toe and heel varied between 35% and 60% of this allowable value. A typical arch roof unit of the Coliseum showed a deflection in the crown of the arch of about 1", which was close to the amount predicted. This deflection may be broken down as the sum of three increments:

A. An elastic arch deflection of about 1", due to dead load, predicted using a modulus of elasticity of 2,000,000 psi as obtained earlier from the deflection of test beams from the same concrete mix.

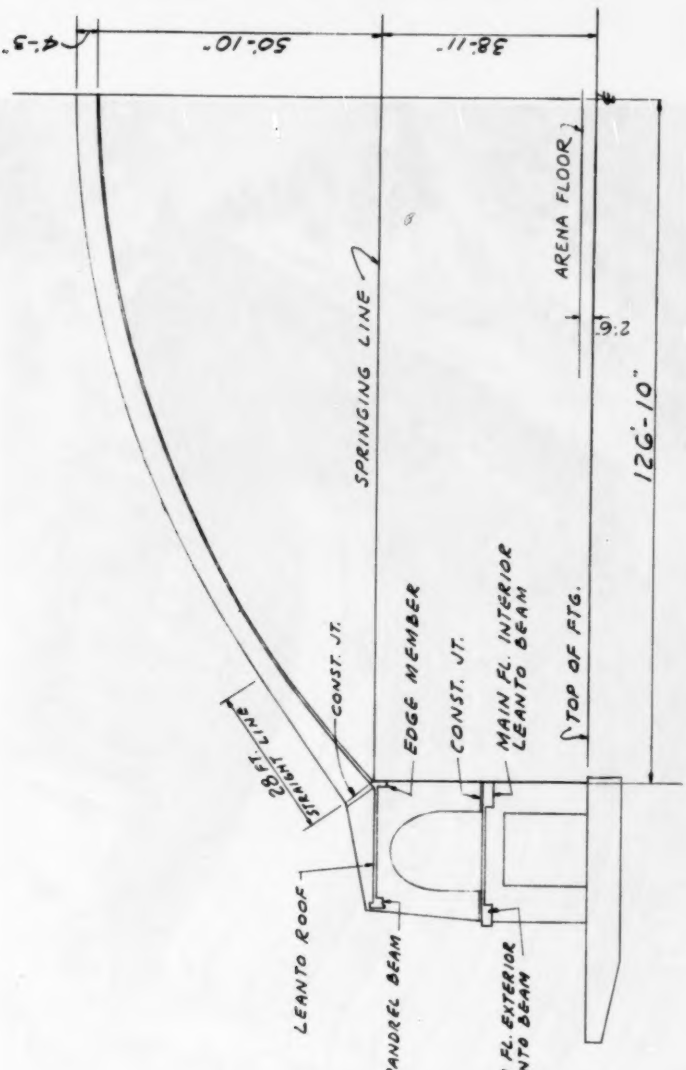
B. A deflection due to the differential between the temperatures at setting and decentering time. A temperature rise of plus 20° resulted in a deflection of minus 7/16".

C. A deflection resulting from the lateral movement of the tops of columns, due to the yielding of the foundations and the flexibility of footings and columns; almost 1/2".

Increment C may be controlled and requires experienced judgment. It was kept under a given maximum value established by optimum economy.

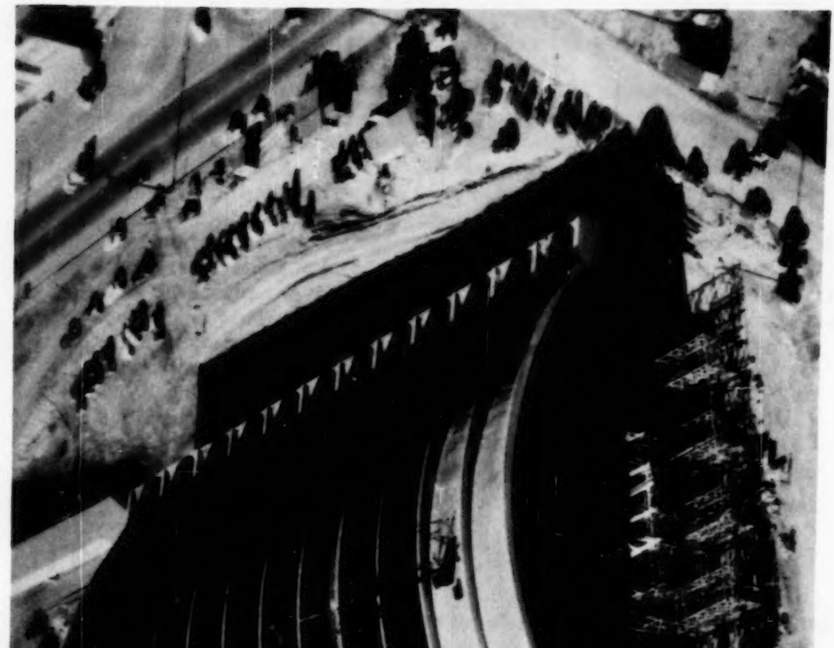
Long, rectangular, slender arch footings had a cost advantage over shorter footings of T-shaped or trapezoidal plan area. The footings are 48' and 49' long. The 7' wide footing, described by Dr. Peck, carries a vertical load of 1440 kips, a horizontal thrust of 460 kips and was designed for a maximum moment of 6600 ft. kips. The corresponding values for the 11' wide footings are: vertical load 2100 kips, horizontal thrust 635 kips and moment 11,670 ft. kips. Figures 3 and 4 indicate loads and soil pressures.

The prediction of Dr. Peck came very close to the values actually measured. It is the possession of such accurate soil information, obtained at reasonable cost, which enables the structural engineer to design economically and safely, close to the conditions later actually encountered in the finished structure.



HALF CROSS SECTION
TYPICAL FOR LINES 6-13

Fig. 1



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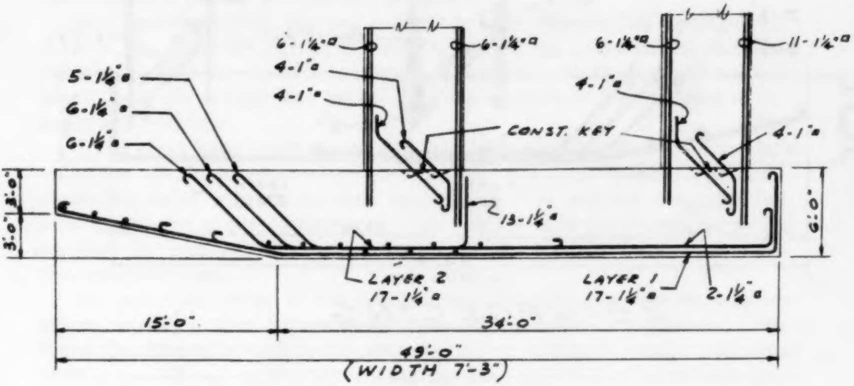

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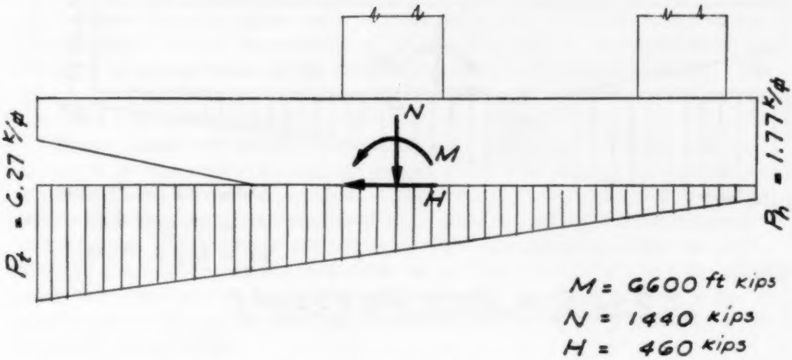
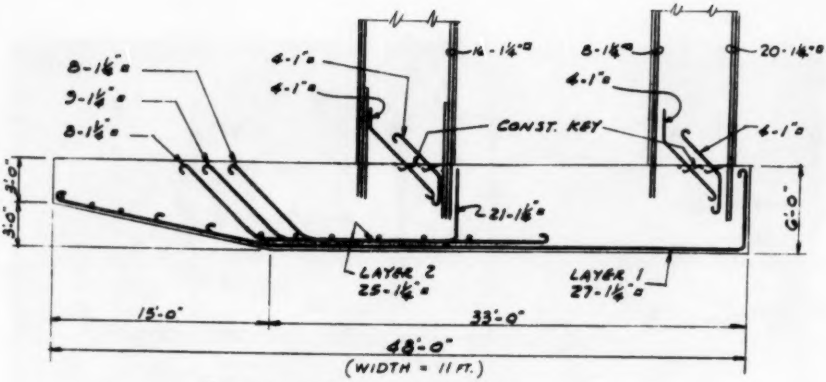


LOADS & SOIL PRESSURE

MAIN ARCH FOOTING (TYPE 1)

Fig. 3



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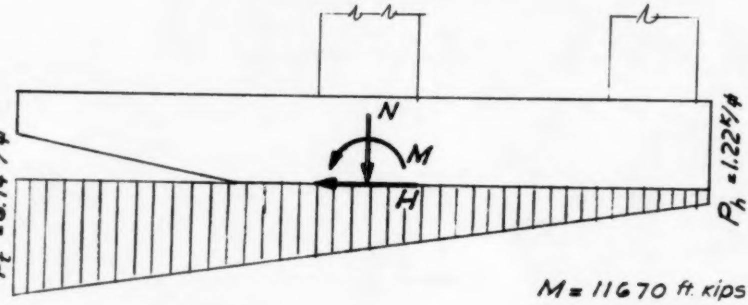

M = 11670 ft. kips
N = 2100 kips
H = 635 kips

LOADS & SOIL PRESSURE

MAIN ARCH FOOTING (TYPE 2)

Fig. 4

RALPH B. PECK,[1] M. ASCE—The discussion by Mr. Tedesko illustrates the advantages of a close relationship between the designers of the superstructure and the investigators of the foundation conditions. The interaction between the superstructure and the subsoil was recognized and was made a basis for the design of the structure. The structural requirements, moreover, dictated to a considerable extent the character of the exploratory program.

Mr. Tedesko points out that a compression beneath the footings of 0.15 in. per 1000 lbs per sq ft was included in the calculations of the structural system. In other words, the subsoil was assumed to be elastic and the design was based upon the concept of the modulus of subgrade reaction.

It has often been pointed out that the determination of an appropriate value for the modulus of subgrade reaction is a most difficult task because the value depends not only upon the kind of soil but also upon the size and shape of the loaded area. Therefore, it is pertinent to inquire whether the application of the concept was justified of the design of the Denver Coliseum.

The principal error in the evaluation of the modulus of subgrade reaction usually arises from the influence of the width of the loaded area. When the subgrade modulus is determined by means of small-scale load tests it is difficult to predict the appropriate values for large footings. In the case of the Coliseum, however, the approximate size of the footings was established at a very early stage in the investigation. The settlement was estimated on the basis of statistical information relating the relative density of the sand to the settlement of loaded areas of various widths. Hence, the method of exploration and its interpretation led directly to an estimate of the settlement of the full-sized footings. The error involved in this procedure was undoubtedly smaller than that associated with extrapolation from small-scale load tests.

These comments indicate that the use of a modulus of subgrade reaction in design requires a close cooperation between the foundation engineer and the structural analyst. There must be a preliminary design with which the foundation engineer may deal in making his estimate of settlements. This design may then require modification after the results of the settlement computations have been introduced into the elastic analysis. Neither the foundation engineer nor the superstructure designer can afford to be ignorant or inappreciative of the problems and methods of the other.

Correction to Proc.-Sep. 326:—On page 326-5, in lines 7 and 16, "Fig. 1" should be "Fig. 2".

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